Receiving Antenna Thermal Noise And Small Signal Amplifier Noise

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Johnson – Nyquist noise, the thermal output noise voltage e_n for a resistor of resistance R_s, is

$$e_n = \sqrt{\{4kTR_sB\}}$$

where $k = 1.37 \times 10^{-23}$ is Boltzmann's constant, T is the absolute temperature (usually taken as 290), and B is the bandwidth in Hertz.

The theory and experimental verification of this formula is found in J. Johnson, "Thermal Agitation of Electricity in Conductors", **Phys. Rev. 32**, 97 (1928) – the experiment, and H. Nyquist, "Thermal Agitation of Electric Charge in Conductors", **Phys. Rev. 32**, 110 (1928) – the theory.

The thermal noise voltage e_n above is the open circuit voltage.

If the thermal noise source resistor R_s is connected to a small signal amplifier with real input impedance R_i, then the thermal noise input voltage E_i to the amplifier is

$$E_i = e_n R_i /(R_i + R_s) = \sqrt{\{4kTR_sB\}}R_i /(R_i + R_s) .$$

This can be regarded approximately as connecting a flag antenna with impedance real part equal to R_s to the amplifier. A better approximation would include an inductor with inductance equal to the flag antenna inductance in series with the source resistance.

When the amplifier input impedance is equal to the resistance of the thermal noise resistor, the input voltage to the amplifier is

$$E_i = \sqrt{\{kT R_i B\}} ,$$

and the input noise power is

$$N_i = E_i{}^2 / R_i = kTB .$$

As can be seen, if the input impedance of the amplifier is real and matched to the thermal noise resistor, then the input noise power is independent of the of the value of the thermal noise resistor. However, note that the input noise voltage to the amplifier, even when the input is matched, is dependent on the value of the thermal noise resistor.

An antenna preamplifier is never matched exactly to the antenna, and so the thermal noise input voltage formula

$$E_i = \sqrt{\{4kTR_sB\}} R_i /(R_i + R_s)$$

would appear to be the correct formula for analyzing the thermal noise of a small receiving antenna. For example, a dummy flag antenna consisting of a resistor with value R_s in series with an inductor of inductance L connected to a preamplifier with all inside a Faraday cage could be used to model the thermal noise of a flag antenna. As the resistor is varied, the noise voltage would be seen to vary, with higher noise voltage values for higher values of R_s and lower noise voltage values for lower values of R_s … at least up to a point.

If R_s is reduced to 0, the observed output voltage of the amplifier is not reduced to 0. The output voltage divided by the gain of the amplifier when $R_s = 0$ is called the equivalent short circuit input noise voltage.

If $R_s \rightarrow \infty$, then $E_i \rightarrow \sqrt{\{4kTR_sB\}}$, from which it may be concluded that the output voltage E_o of a follower is

$$E_o = c\sqrt{\{4kTR_sB\}}$$

where c is a constant somewhat less than 1. Again it is found that if R_s is reduced to 0, the observed output voltage of the amplifier is not reduced to 0, but is asymptotic to the equivalent short circuit output noise voltage. These cases are of interest because have demonstrated that the noise output of a low noise follower such as the

complementary J310 – J271 FET follower used as PPL's for some of the flag antennas in The Dallas Files is dominated by the thermal noise of the flag terminating resistor, and not by noise originating in the follower and that the equivalent short circuit input noise voltage of the FET follower is less than the equivalent short circuit input noise voltage of a low noise Norton transformer feedback amplifier.

If no variation in output voltage of the amplifier as described above is observed for values of R_s in and around typical values of flag antenna terminating resistors (500 to 1000 ohms), then the thermal noise due to R_s is not the dominant noise in the noise output of the preamplifier. This may happen, for example, when the input of the amplifier is a high Q LC tuned circuit with link input coupling. As an approximation to a dummy flag with 1000 ohm termination, 3:1 turns ratio step down broadband transformer (matching to approximately 100 ohms), and a 40 turn high Q LC circuit tuned to 2000 kHz with 4 turn link, let us consider a 40 turn high Q LC tuned circuit with 4 turn link and high input impedance follower connected across the LC tuned circuit, with a 100 Ω source resistor connected across the link, and let us assume the loaded Q is 150. The impedance z_0 at resonance is

$$z_0 = Q\omega_0 L - j/(\omega_0 C)$$

so that the thermal noise resistance at resonance is $300\pi L \times 10^6$. If we take L = 20 μH, then the thermal noise resistance is 18,850 ohms. Thus the thermal noise voltage across the parallel LC tuned circuit due to the parallel LC tuned circuit is about

$$E_{LC} = 275\sqrt{\{kTB\}} \text{ volts.}$$

The above is a variation of a result in Belrose's classic article "Ferromagnetic Loop Aerials," **Wireless Engineer**, February 1955, 41– 46

The thermal noise voltage due to the flag antenna is equivalent to a 100 ohm resistor across the 4 turn link. It follows

$$E_i = \sqrt{\{4kTx100xB\}}188 /(188 + 100) = 13\sqrt{\{kTB\}}$$

so that the equivalent thermal noise voltage due to the flag antenna across the LC tuned circuit is about

$$E_{flag} = 130\sqrt{\{kTB\}} \text{ volts}$$

due to the 1:10 link turns ratio.

Thus it is seen that the thermal noise due to the parallel LC tuned circuit masks the thermal noise due to the 1000 ohm flag terminating resistor. For a smaller flag terminating resistor, a higher loaded Q, or a larger LC tuned circuit inductance the masking would be more pronounced.

It has been proposed that inserting a high Q LC tuned circuit with link input and parallel LC tuned circuit connected to a FET follower between the lead in and receiver at the receiver will improve weak signal performance of flag antennas and flag antenna arrays. A dummy antenna model of the single antenna case (sans lead in) would be a thermal noise source resistor (the flag terminating resistor) in series with an inductor (the flag loop inductance), across the primary of a 3:1 turns ratio broadband step down transformer with secondary connected to the link input of the LC tuned circuit, with the parallel LC tuned circuit connected to a FET follower. When the resistance of the thermal noise source resistance was reduced to 0, the noise output power actually increased by a few dB. This was contrary to the expected decrease in noise output power as the thermal noise source resistance was reduced to 0. The noise output power of the dummy antenna simulation with 1000 ohm thermal noise source resistance and 3:1 step down transformer was also greater than when a 100 ohm resistor only (no 3:1 step down transformer) was connected across the link input of the LC tuned circuit; in this case the noise output power did decrease as the 100 ohm resistance was decreased to 0. When lead in was included in the 3:1 step down transformer model, output noise power increased a few dB as the thermal noise source resistance was reduced to 0. More thorough theoretical analyses of these cases appears to be in order. PSpice simulations were inconclusive. PSpice noise amplitude increases and decreases followed the same pattern, but PSpice noise amplitudes were quite different. Perhaps the PSpice models I used were not correct.